

**SCORING – DO NOT REMOVE THIS SHEET FROM THE FRONT OF THE EXAM**

Section 1: 47 / 50

Section 2: 47 / 60

Section 3: 40 / 40

Section 4: 33 / 50

Section 5: 31 / 31

Section 6: 28 / 28

Total: 226 / 259

*great job!*

# PERIODIC TABLE OF THE ELEMENTS

<http://www.kjf-split.hr/periodnt/en/>

GROUP	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
GROUP	IA	IIA	IIIB	IVB	VB	VIB	VII	VIII	IX	X	XI	XII	IIIA	IVA	VA	VIA	VIIA	VIIIA
PERIOD	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	H 1.0079																	He 4.0026
2	Li 6.941	Be 9.0122											B 10.811	C 12.011	N 14.007	O 15.999	F 18.998	Ne 20.180
3	Na 22.990	Mg 24.305	Al 26.982	Si 28.086	P 30.974	S 32.065	Cl 35.453	Ar 39.948					B 10.811	C 12.011	N 14.007	O 15.999	F 18.998	Ne 20.180
4	K 39.098	Ca 40.078	Sc 44.956	Ti 47.867	V 50.942	Cr 51.996	Mn 54.938	Fe 55.845	Co 58.933	Ni 58.693	Cu 63.546	Zn 65.39	Al 26.982	Si 28.086	P 30.974	S 32.065	Cl 35.453	Ar 39.948
5	Rb 85.468	Sr 87.62	Y 88.906	Zr 91.224	Nb 92.906	Mo 95.94	Tc (98)	Ru 101.07	Rh 102.91	Pd 106.42	Ag 107.87	Cd 112.41	Al 26.982	Si 28.086	P 30.974	S 32.065	Cl 35.453	Ar 39.948
6	Cs 132.91	Ba 137.33	La-Lu 57-71	Hf 178.49	Ta 180.95	W 183.84	Re 186.21	Os 190.23	Ir 192.22	Pt 195.08	Au 196.97	Hg 200.59	Al 26.982	Si 28.086	P 30.974	S 32.065	Cl 35.453	Ar 39.948
7	Fr (223)	Ra (226)	Ac-Lr 89-103	Rf (261)	Db (262)	Sg (266)	Bh (264)	Hs (277)	Mt (268)	Uu (281)	Uu (272)	Uub (285)	Al 26.982	Si 28.086	P 30.974	S 32.065	Cl 35.453	Ar 39.948
													Uuq (289)					

## LANTHANIDE

57	58	59	60	61	62	63	64	65	66	67	68	69	70	71
La	Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu
57 138.91	58 140.12	59 140.91	60 144.24	61 (145)	62 150.36	63 151.96	64 157.25	65 158.93	66 162.50	67 164.93	68 167.26	69 168.93	70 173.04	71 174.97

## ACTINIDE

89	90	91	92	93	94	95	96	97	98	99	100	101	102	103
Ac	Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr
89 (227)	90 232.04	91 231.04	92 238.03	93 (237)	94 (244)	95 (243)	96 (247)	97 (247)	98 (251)	99 (252)	100 (257)	101 (258)	102 (259)	103 (262)

(1) Pure Appl. Chem., 73, No. 4, 667-683 (2001)  
 Relative atomic mass is shown with five significant figures. For elements having no stable nuclides, the value enclosed in brackets indicates the mass number of the longest-lived isotope of the element.  
 However three such elements (Th, Pa, and U) do have a characteristic terrestrial isotopic composition, and for these an atomic weight is tabulated.  
 Editor: Aditya Vardhan (adv@netlinx.com)

## Values of Some Physical Constants

Constant	Symbol	Value
Atomic mass constant	$amu$	$1.660\ 5402 \times 10^{-27}$ kg
Avogadro constant	$N_A$	$6.022\ 1367 \times 10^{23}$ mol <sup>-1</sup>
Bohr magneton	$\mu_B = eh/2m_e$	$9.274\ 0154 \times 10^{-24}$ J · T <sup>-1</sup>
Bohr radius	$a_0 = 4\pi\epsilon_0^2/r_e e^2$	$5.291\ 772\ 49 \times 10^{-11}$ m
Boltzmann constant	$k_B$	$1.380\ 658 \times 10^{-23}$ J · K <sup>-1</sup> $0.695\ 038$ cm <sup>-1</sup>
Electron rest mass	$m_e$	$9.109\ 3897 \times 10^{-31}$ kg
Gravitational constant	$G$	$6.672\ 59 \times 10^{-11}$ · m <sup>3</sup> · kg <sup>-1</sup> · s <sup>-2</sup>
Molar gas constant	$R$	$8.3145101$ J · K <sup>-1</sup> · mol <sup>-1</sup> $0.083\ 1451$ dm <sup>3</sup> · bar K <sup>-1</sup> · mol <sup>-1</sup> $0.082\ 0578$ dm <sup>3</sup> · atm K <sup>-1</sup> · mol <sup>-1</sup>
Molar volume, ideal gas (one bar, 0°C)		$22.711\ 08$ L · mol <sup>-1</sup>
(one atm, 0°C)		$22.414\ 09$ L · mol <sup>-1</sup>
Nuclear magneton	$\mu_N = e\hbar/2m_p$	$5.050\ 7866 \times 10^{-27}$ J · T <sup>-1</sup>
Permittivity of vacuum	$\epsilon_0$	$8.854\ 187\ 816 \times 10^{-12}$ C <sup>2</sup> · J <sup>-1</sup> · m <sup>-1</sup>
	$4\pi\epsilon_0$	$1.112\ 650\ 056 \times 10^{-10}$ C <sup>2</sup> · J <sup>-1</sup> · m <sup>-1</sup>
Planck constant	$h$	$6.626\ 0755 \times 10^{-34}$ J · s
	$\hbar$	$1.054\ 572\ 66 \times 10^{-34}$ J · s
Proton charge	$e$	$1.602\ 177\ 33 \times 10^{-19}$ C
Proton magnetogyric ratio	$\gamma_p$	$2.675\ 221\ 28 \times 10^8$ s <sup>-1</sup> · T <sup>-1</sup>
Proton rest mass	$m_p$	$1.672\ 6231 \times 10^{-27}$ kg
Rydberg constant (Bohr)	$R_\infty = m_e e^4 / 8\epsilon_0^2 h^2$	$2.179\ 8736 \times 10^{-23}$ J $109\ 737.31534$ cm <sup>-1</sup>
Rydberg constant for H	$R_H$	$109677.581$ cm <sup>-1</sup>
Speed of light in vacuum	$c$	$299\ 792\ 458$ m · s <sup>-1</sup> (defined)
Stefan-Boltzmann constant	$\sigma = 2\pi^5 k_B^4 / 15h^3 c^2$	$5.670\ 51 \times 10^{-8}$ J · m <sup>-2</sup> · K <sup>-4</sup> · s <sup>-1</sup>

**Section 1. (50 pts) Glowstick Kinetics (you will have up to 25 minutes to work on the lab portion but you may perform calculations and work later)**

**Materials**

3 glow sticks (4" long, ~10 mm diameter)

~12x12" piece of Al foil

tongs

thermometer

3 beakers

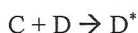
ice

hot water bath (set to ~40°C)

Labquest Spectrometer

**Goal:** Using the above equipment, determine the activation energy  $E_a$  for the lightstick reaction.

A simplified version of the glowstick reaction is:



Since photons ( $h\nu$ ) are a "product" one can measure reaction *rate* directly by measuring the intensity (photons/time/area) coming from the lightstick.

**Instructions /tips:**

0) Read the whole problem even if you can't do all parts

1) You **MUST** set your Labquest pro to *intensity mode*. To do this tap on the USB: \_\_\_ screen, then select Intensity (see photo at right, if there is a problem, ask your TA for help)

2) You can measure the intensity vs. wavelength spectrum of the glow stick by placing it into the cuvette holder after pressing the "play" button

3) What wavelength do you want to measure the light at?

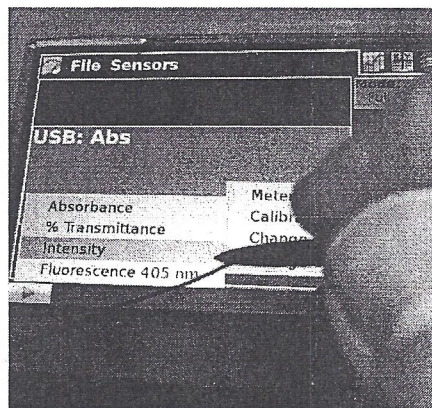
4) do you need to (manually) subtract a background?

600

5) make sure you shake your glowstick so that there are no bubbles at the bottom where you are measuring the light from

6) Start your lightsticks **ALL** at the same time. Let them run for 5-10 minutes before using them. Cycle through them. They have a fast initial decay so if you wait a long time between measuring one stick and another the difference will be complicated by their decay in time.

7) Do **NOT** turn the hot plates up past 40 C. The higher temperatures will damage your light sticks.



Section 1. Glow sticks Data table (for convenience only)

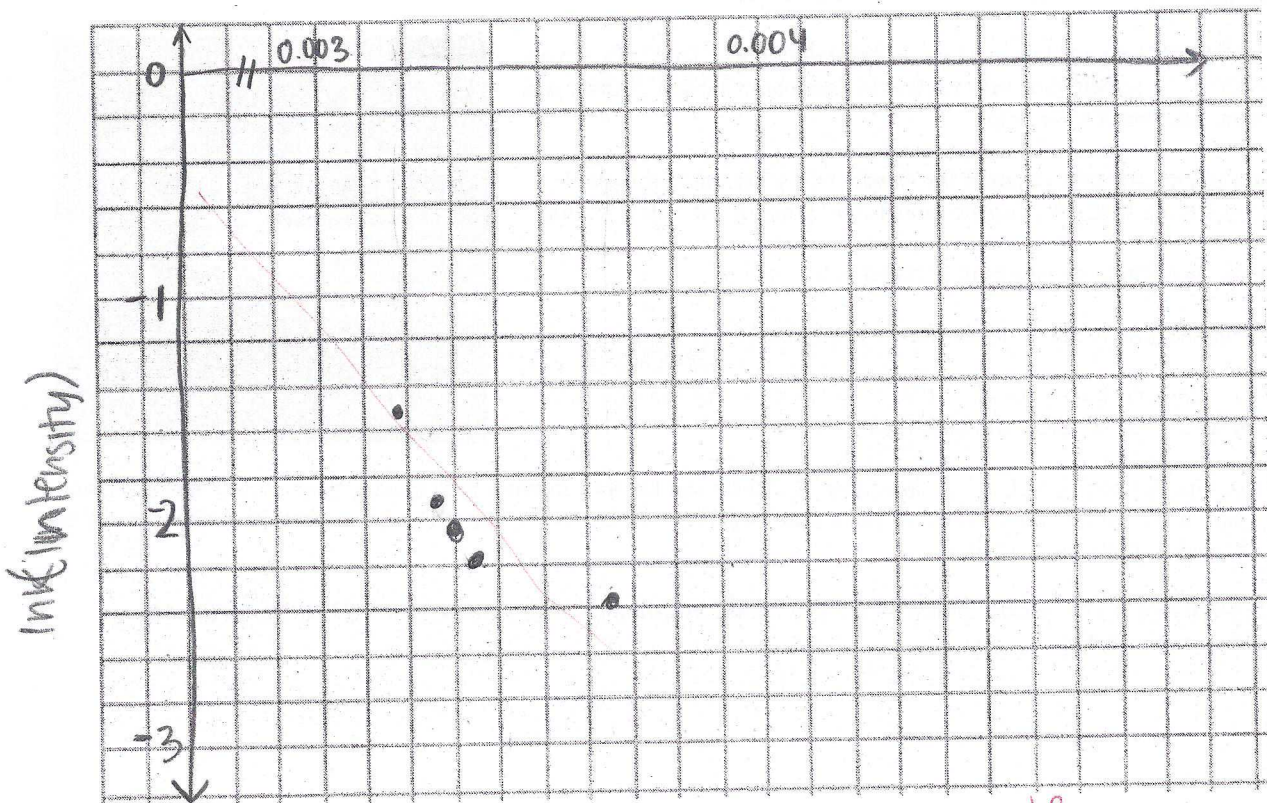
T (C)	Intensity	1/T (K)	ln(Intensity)
40.0	0.215 rel	0.00319	-1.537
34.5	0.139 rel	0.00325	-1.973
31.6	0.129	0.00328	-2.048
0	0.095	0.00306	-2.354
29.1	0.117	0.00335	-2.15

(A) (5 pts) Write down the Arrhenius equation relating activation energy, temperature and reaction rate:

$$k = A e^{-E_a/RT} \quad \therefore \ln k = \ln A - \frac{E_a}{R} \left(\frac{1}{T}\right)$$

(B) (25 pts) Plot your data below in order to determine  $E_a$  in kJ/mol ( $R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$ ). Label your axes. Show your work.

Describe what you are plotting 1/T vs. ln(Intensity) (5 pts)



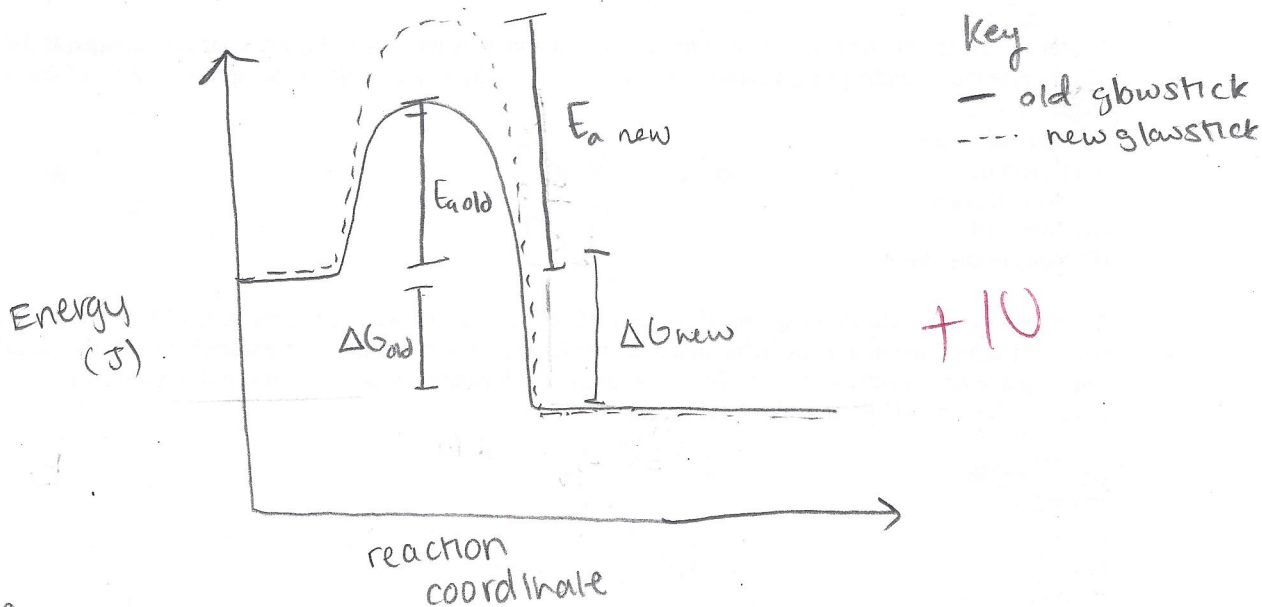
slope =  $-2500 \text{ K}^{-1}$

$E_a = 300.7 \text{ J/mol}$

$$-\left(\frac{-2500 \text{ K}^{-1}}{8.314 \text{ J/molK}}\right)$$

check  $m = \frac{\Delta y}{\Delta x} = \frac{-2.048 + 1.973}{0.00328 - 0.00325 \text{ K}^{-1}} = -2500$

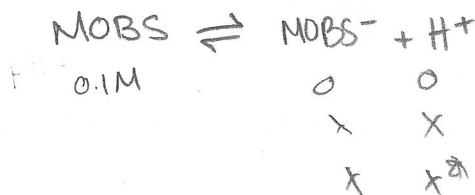
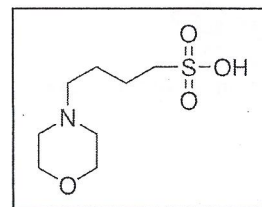
1C) (10 Points) A new glowstick reaction is found that is more sensitive to variations in light emission with temperature. Sketch 2 plots of energy vs. reaction coordinate for these two reactions one labeled "New glow stick" and one labeled "old glowstick", label  $\Delta G$  and  $E_a$  on both plots.



#  $K = Ae^{-E_a/RT}$   
 more sensitive  
 to var in  
 temp = higher  
 $E_a$

$$E_{a\text{ new}} > E_{a\text{ old}}$$

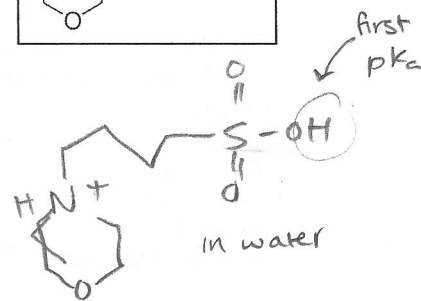
1D) (10 points) The structure of MOBS is shown at right. Calculate the pH of a 0.1M MOBS solution in water, given the MOBS  $pK_a = 7.6$ . Yeah redemption points!



$$\frac{x^2}{0.1-x} = 10^{-7.6}$$

$$x = 5.01 \times 10^{-5} \text{ M}$$

+10



$$-\log(5.01 \times 10^{-5}) = \boxed{4.3}$$

that seems  
 slightly too  
 acidic?  
 even if  $pK_{a1} = 2.2$  ish or  
 whatever

**Section 2. (60 pts) Multiple Choice – WRITE YOUR ANSWER IN THE BOX**  
 (-1 point for an incorrect guess on any problem) (4 points each)  
 More GRE-like and MCAT-like questions to help you practice

$$\text{rate} = \frac{V[S]}{K_m + [S]}$$

1) Many enzyme reactions follow the rate law shown above, where  $V$  and  $K_m$  are constants and  $[S]$  is the concentration of substrate that is undergoing a catalyzed reaction. When  $[S] \gg K_m$ , what is the apparent order of the reaction?

- (A) Zero order
- (B) One-half order
- (C) First order
- (D) Second order
- (E) Third order
- (F) None of the above

$$\text{rate} = \frac{V[S]}{[S]} = V$$

1) A ✓

2)  $^{123}\text{I}$  decays via electron capture with a half-life of 13.22h followed by emission of 159 keV and 27 keV gamma rays.  $^{123}\text{I}$  is used in nuclear medicine imaging supplied as sodium iodide. The packaging reads that at calibration time, each capsule has an activity of 3.7 MBq, (100  $\mu\text{Ci}$ ). Each gelatin capsule contains not more than 1 g of sucrose. What mass of  $^{123}\text{I}$  is in a single capsule at calibration time?

- (A) 97,000 fg
- (B) 52,000 fg
- (C) 5.2 pg
- (D) 9.7 mg
- (E) 9.7  $\mu\text{g}$
- (F) none of the above

$$3.7 \text{E}6 \text{Bq} = \lambda N$$

$$\frac{3.7 \text{E}6 \text{Bq}}{\ln 2 / 13.22 \text{h}} = N = 2.54 \text{E}11$$

$$2.54 \text{E}11 \cdot \frac{1 \text{mol}}{6.02 \text{E}23} \cdot \frac{123 \text{g}}{1 \text{mol}} = 5.19 \text{E}-11 \text{g} \cdot 10^6 = 52 \text{pg}$$

2) B ✓

3) In a 0.1M solution of a weak acid,  $[A^-] = [HA]$  at  $[H^+] = 3.47 \times 10^{-5}$ , the  $pK_a$  of this acid is closest to:

- (A) 1.5
- (B) 2.5
- (C) 3.5
- (D) 4.5
- (E) 5.5
- (F) none of the above

$$K_a = \frac{[H^+][A^-]}{[HA]} = 3.47 \text{E}-5$$

3) D ✓

4) A star with a surface temperature of 6000K emits photons of many energies. What is the energy of red photons with  $\lambda = 620 \text{ nm}$ , being emitted by the star (in eV)?

- (A) 3.0
- (B) 2.0
- (C) 1.0
- (D)  $3.2 \times 10^{-19}$
- (E)  $9.9 \times 10^{-17}$
- (F) none of the above

$$E = hc/\lambda = \frac{1240 \text{ eV nm}}{620 \text{ nm}} = 2$$

4) B ✓

5)  $^{11}\text{C}$  decays by positron emission to form  $^{11}\text{B}$ . If the mass of an  $^{11}\text{B}$  atom is 11.0093055 amu and the mass of an  $^{11}\text{C}$  atom is 11.011433 amu and the electron mass is 0.00054858 amu, and  $1 \text{ amu} = 931.5 \text{ MeV}/c^2$ , then maximum KE of the positron emitted is closest to:

- (A) 0.449 MeV
- (B) 0.960 MeV
- (C) 1.47 MeV
- (D) 1.98 MeV
- (E) none of the above



$$E = \Delta mc^2 = \left( (11.0093055) + 2(0.00054858) - 11.011433 \right) \cdot 931.5 \text{ MeV}/c^2 = 0.960 \text{ MeV}$$

5) B ✓

$$-0.00103 \cdot 931.5$$

- 6) An electron is  
 (A) made from quarks  
 (B) made from neutrinos  
 (C) a meson  
 (D) a baryon  
 (E) a lepton  
 (F) a leprechaun yes can't see it  
 (G) none of the above

6) E ✓

7) A radioactive isotope of Copper,  $^{64}\text{Cu}$ , decays via beta emission to form  $^{64}\text{Zn}$  with a half-life of 12.8 hours. Starting with 200g of  $^{64}\text{Cu}$ , how much  $^{64}\text{Zn}$  is produced in 25.6 hours?

- (A) 200.0 g  
 (B) 150.0 g  
 (C) 138.6 g  
 (D) 100.0 g  
 (E) 40.0 g  
 (F) 25.0 g  
 (G) none of the above

2 half lives  

$$\frac{200}{2} = 100 + \frac{100}{2} = 150$$

7) B ✓

8)  $^{99\text{m}}\text{Tc}$  is a radioisotope used in nuclear medicine. It decays to  $^{99}\text{Tc}$  by emission of 140 keV gamma rays with a half life of 6.0 hours. If the initial activity of a sample is 200 MBq, what is the momentum of a photon being emitted 12 hours later?

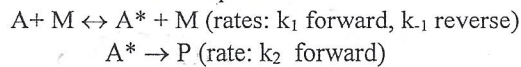
- (A)  $2.24 \times 10^{-14}$  J  
 (B)  $2.24 \times 10^{-14}$  kg m/s  
 (C)  $7.46 \times 10^{-23}$  kg m/s  
 (D)  $8.85 \times 10^{-3}$  kg m/s  
 (E) 50 MBq  
 (F) 35 keV  
 (G) None of the above

$E = pc$

$$\frac{E}{c} = p = \frac{140 \text{ BeV}}{3 \times 10^8 \text{ m/s}} \cdot \frac{1.602 \times 10^{-19} \text{ J}}{1 \text{ eV}} =$$

8) C ✓

9) Given the following mechanism in which neither step can be considered rate limiting:



The rate expression is best given by:

- (A)  $d[P]/dt = -k_1 k_2 [A][M] / k_{-1} [M]$   
 (B)  $d[P]/dt = k_1 [A][M] / (k_{-1} [M] + k_1)$   
 (C)  $d[P]/dt = k_1 k_2 [A][M] / (k_{-1} [M] + k_2)$   
 (D)  $d[P]/dt = k_1 [A][M]$   
 (E)  $d[P]/dt = k_1 k_2 [A] / [M]$   
 (F) none of the above

$$[A^*] = \frac{[A][M]k_1 - [A^*][M]k_{-1} - [A^*]k_2}{k_{-1}[M] + k_2}$$
  

$$[A^*] = \frac{k_1 [A][M]}{k_{-1}[M] + k_2}$$
  

$$\frac{d[P]}{dt} =$$

9) C ✓

10) An electron has an uncertainty in its position of  $1 \text{ \AA}$ . What is the minimum uncertainty the momentum of this electron?

- (A)  $6.63 \times 10^{-24}$   
 (B)  $3.31 \times 10^{-24} \text{ J}$   
 (C)  $5.3 \times 10^{-25} \text{ J}$   
 (D)  $5.3 \times 10^{-25}$   
 (E) None of the above

$\Delta x \Delta p \geq \frac{h}{4\pi}$   
 $\Delta p \geq \frac{6.626 \text{ Js} \cdot 10^{-34}}{4\pi (1 \text{ \AA} \cdot \frac{1 \text{ m}}{10^{10} \text{ \AA}})} \geq 5.27 \times 10^{-25} \text{ kg m/s}$

10)

$h$  is in units of momentum

$\Delta p \geq \frac{h}{4\pi \Delta x}$

$5.27 \times 10^{-25} \text{ kg m/s}$



11) The function  $f(x,t) = \sin(kx)\cos(\omega t)$  is an example of a:

- (A) function I never want to see again *yes*
- (B) wave travelling in the +x direction (to the right)
- (C) classical 1D standing wave
- (D) quantum mechanical 1D standing wave
- (E) wave travelling in the -x direction (to the left)

11) C ✓

12) The Bohr model explains the Rydberg equation, and predicts the UV-visible emission spectrum of the hydrogen atom. A form of the Bohr model / Rydberg equation may also be used to predict the UV-visible emission for all of the following EXCEPT:

- (A) helium cation,  $\text{He}^+$  ✓
- (B) deuterium atom,  $\text{D}$   ${}^2\text{H}^+$  ✓
- (C) beryllium cation,  $\text{Be}^{3+}$  ✓
- (D) hydride ion,  $\text{H}^-$  ✓
- (E) tritium atom *what is this*
- (F) All of the above can be predicted

*single electron*

12)

13) If  $z = 3 + 5i$ , then  $z^* z =$

- (A) 8
- (B) -16
- (C) +16
- (D) 34
- (E) 3
- (F) none of the above

$$(3+5i)(3-5i)$$

$$9 + (25)i^2 = -16$$

*but always positive!*

13) C ✗

14) An electron is confined to a 1D box with infinitely high walls. If the box was 1 nm wide, the electron could absorb photons of energy:

- (A) 0.377 eV
- (B) 1.13 eV
- (C)  $1.13 \times 10^{-9}$  eV
- (D) 1.77 eV
- (E) None of the above

$$E_n = \frac{n^2 h^2}{8 m L^2} = 6.02 \times 10^{-20} \text{ J}$$

$$= 0.377 \text{ eV}$$

14) A ✓

15) If a wavefunction  $\psi(x)$  is normalized, then it means that:

- (A)  $\psi^*(x) \psi(x) = 1$
- (B)  $\psi^*(x) \psi(x) = 0$
- (C)  $\int_{-\infty}^{\infty} \psi^*(x) \psi(x) dx = 0$
- (D)  $\int_{-\infty}^{\infty} \psi^2(x) dx = 1$
- (E)  $\int_{-\infty}^{\infty} \psi^*(x) \psi(x) dx = 1$
- (F) none of the above

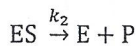
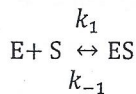
15) E ✓

*12(4) - 1 = 47*

Section 3. (40 pts) Catalysis, Enzymes, and the Steady State Approximation

3. Kinetics

The classic mechanism for an enzyme catalyzed reaction is:



Where E is the free enzyme, S is the substrate, and P is the product.

(3.A.1) (5 pts) State the key assumption of the steady state approximation:

$$\frac{d[ES]}{dt} = 0 \quad \text{no rate-limiting step}$$

(3.A.2) (10 pts) Apply the steady state approximation to this mechanism to derive the expression for the rate of product formation,  $d[P]/dt$  in terms of substrate concentration  $[S]$ , total enzyme concentration  $[E_T]$ , and the Michaelis-Menten constant,  $K_m = (k_{-1} + k_2) / k_1$ . Show all work. Circle your key steps (especially what  $d[ES]/dt = ?$ .)

$$\frac{d[ES]}{dt} = k_1[E][S] - k_{-1}[ES] - k_2[ES] = 0$$

$$[E_T] = [E] + [ES] \quad \therefore [E_T] - [ES] = [E]$$

$$\frac{d[ES]}{dt} = 0 = k_1[E_T][S] - k_{-1}[ES][S] - k_2[ES]$$

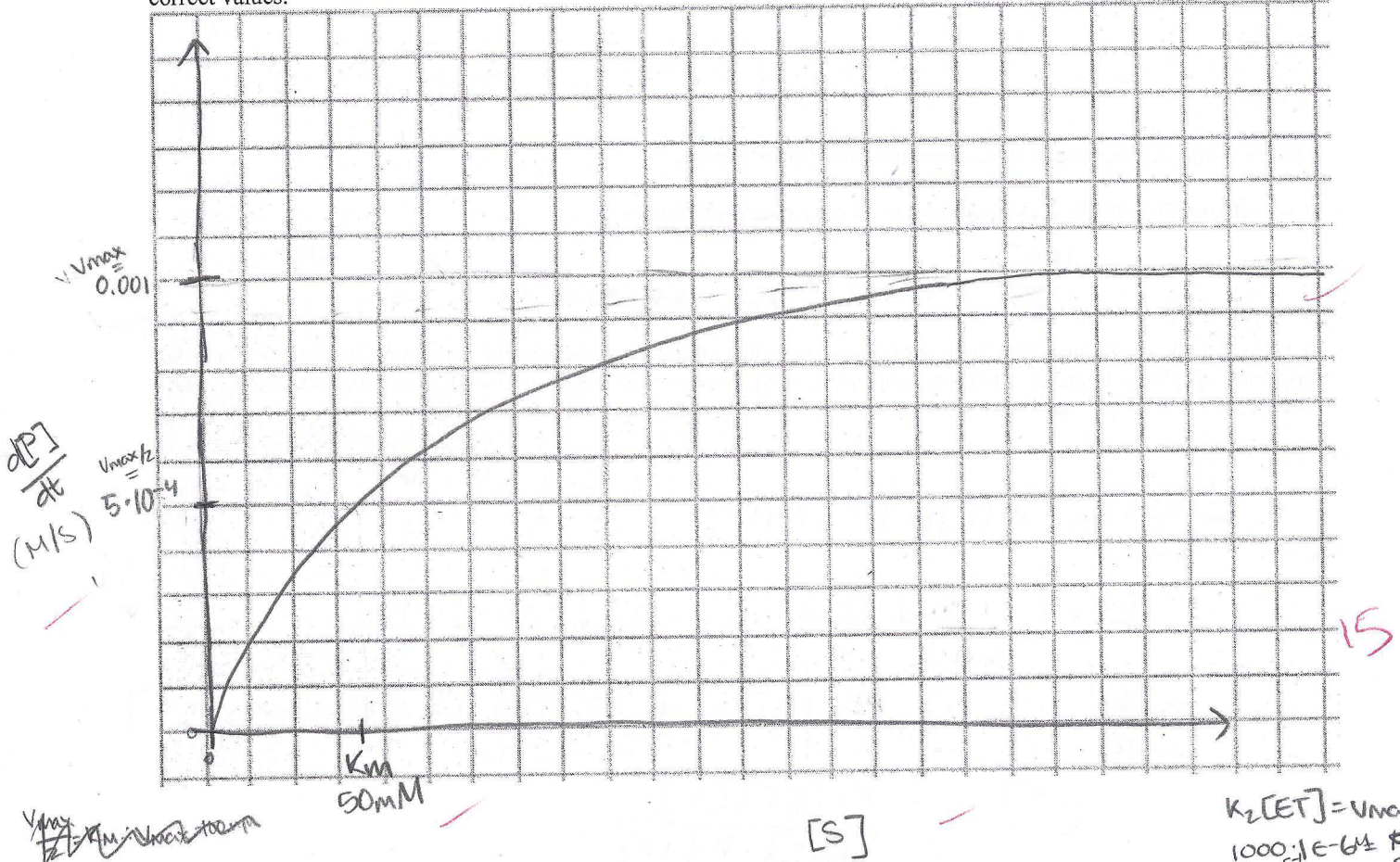
$$[ES] = \frac{k_1[E_T][S]}{k_1[S] + k_{-1} + k_2}$$

$$\frac{d[P]}{dt} = k_2[ES] = \frac{k_1 k_2 [E_T][S]}{k_1[S] + k_{-1} + k_2} \left( \frac{1/k_1}{1/k_1} \right)$$

note:  $\frac{k_{-1} + k_2}{k_1} = K_m$   
here!

$$\frac{d[P]}{dt} = \frac{K_2 [E_T][S]}{K_m + [S]}$$

(3.B) (15 pts) Glucose oxidase is an enzyme that catalyzes the oxidation of beta-D-glucose to hydrogen peroxide and D-glucono-δ-lactone. It is widely used in diagnostic assays, such as tests for blood sugar levels. It has an approximate  $K_m = 50 \text{ mM}$ , and  $k_{cat} = k_2 = 1000 \text{ s}^{-1}$ . On the graph below, sketch  $d[P]/dt$  vs.  $[S]$ , for the entire interesting range of behavior if the total concentration of glucose oxidase is  $1 \times 10^{-6}$  molar. Label  $V_{max}$  and  $K_m$  and make sure they fall at the correct values.



(3C) (5 pts) Under what conditions will the oxidation of glucose substrate by the glucose oxidase enzyme appear to be zeroth order in [glucose]? (use equations for full credit)

when  $[S] \gg K_m$

$$\frac{d[P]}{dt} = \frac{k_2 [S][E_T]}{[S] + k_m}$$

when  $[S] \gg K_m$  or  $[S] \gg 50 \text{ mM}$  because then  $\frac{d[P]}{dt} = k_2 [E_T] = 0.001 \text{ M/s}$

$$\frac{d[P]}{dt} = \frac{k_2 [S][E_T]}{[S] + 0 \text{ ish}}$$

$k_2 [E_T] = V_{max} = 1000 \cdot 1 \times 10^{-6} = 0.001 \text{ M/s}$

(3D) (5 pts) Under what conditions will the oxidation of glucose substrate by the glucose oxidase enzyme appear to be first order in [glucose]? (use equations for full credit)

first order in glucose when  $K_m \gg [S]$  or  $50 \text{ mM} \gg [S]$

$$\frac{d[P]}{dt} = \frac{k_2}{K_m} [S][E_T] \text{ then } \frac{d[P]}{dt} = (0.02 \text{ s}^{-1} \text{ M}^{-1}) [S]$$

because  $\frac{d[P]}{dt} = \frac{k_2 [S][E_T]}{K_m + [S]} \rightarrow$  basically

**Section 4. (50 points) Nuclear**

Approved in 2013,  $^{223}\text{RaCl}_2$  (FW=296.81 g/mol) became the first alpha emitter ever to be licensed for use as a radiopharmaceutical in the United States and is now marketed under the trade name Xofigo. As a calcium mimetic,  $^{223}\text{Ra}^{2+}$  is taken up preferentially in regions of osteoblastic metastasis (bone tumors) and has been shown to improve survival rates and quality of life for castration resistant prostate cancer patients with symptomatic bone metastases.  $^{223}\text{Ra}$  decays via alpha emission with a half-life of 11.4 days.

(A) (5 pts) In 1-2 sentences, explain why an alpha emitter such as  $^{223}\text{Ra}$  might be preferable to a beta or gamma emitter for radiation therapy if the drug can be delivered specifically to the site of the cancer.

Alpha particles will travel less far into the tissue than beta or gamma rays and cause less tissue damage. It is heavier and less able to travel through tissue.

(B) (10 pts) The recommended dose of  $^{223}\text{RaCl}_2$  is 50 kBq/kg (1 Bq = 1 decay / s), and is supplied as standardized solutions buffered for intravenous injection with 1000 kBq/ml activity at calibration time. What volume of solution (in mL) should you inject into a patient weighing 60 kg if it is now 2.0 days after the calibration date of your  $^{223}\text{RaCl}_2$  solution?

$$A = 1000 \frac{\text{Bq}}{\text{ml}} \cdot e^{-\ln(2)/11.4 \cdot 2} = 885498 \text{ Bq/ml}$$

$$\frac{1 \text{ ml}}{885498 \frac{\text{Bq}}{\text{ml}}} \cdot \frac{50 \text{ kBq}}{\text{kg}} \cdot 60 \text{ kg} = 0.003387 \text{ mL}$$

volume = 0.00339 mL

$A = \frac{A_0}{e^{kt}}$

$1000 \cdot e^{-\ln(2)/11.4 \cdot 2} = A_f$

$A_f \cdot \frac{1 \text{ ml}}{885498 \text{ Bq/ml}} \cdot \frac{50 \text{ kBq}}{\text{kg}} \cdot 60 \text{ kg} = \text{mL}$

(C) (10 points) The biological half-life represents the time it takes the body to clear a compound metabolically. If the biological half-life for  $\text{Ra}^{2+}$  is 3.5 days, and the radioactive half-life is 11.4 days, calculate the activity of  $^{223}\text{Ra}$  remaining in the 60 kg patient dosed with 50 kBq/kg 5 days after treatment.

$k_{\text{bio}} = 0.198 \text{ days}^{-1}$   
 $k_{\text{rad}} = 0.0608 \text{ days}^{-1}$   
 $k_{\text{total}} = 0.259 \text{ days}^{-1}$

activity = 822.3 kBq

$\frac{50 \text{ kBq}}{\text{kg}} \cdot 60 \text{ kg} \cdot \frac{1000 \text{ Bq}}{1 \text{ kBq}} \cdot e^{-0.259 \cdot 5} = A = 822332.9 \text{ Bq} = 822 \text{ kBq}$

$\frac{50 \text{ kBq}}{\text{kg}} \cdot 60 \text{ kg} \cdot \frac{1000 \text{ Bq}}{1 \text{ kBq}} = A_0$   
 $A_0 = kN$   
 $N = 151515$

$151515 e^{-0.259 \cdot 5} = N_f = 414996.7$

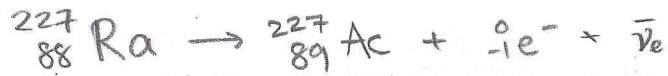
$A = 414996.7 \cdot 0.198$   
 $A = 822 \text{ kBq}$  idk same answer

activity should only take  $k_{\text{rad}}$  into account? idk

25

(D)  $^{223}\text{Ra}$  is produced by neutron bombardment of  $^{226}\text{Ra}$  to produce  $^{227}\text{Ra}$ . The  $^{227}\text{Ra}$  then decays to  $^{227}\text{Ac}$  with a half-life of 47 min.  $^{227}\text{Ac}$  decays to  $^{227}\text{Th}$  with a half-life of 21.8 years, and the resulting  $^{227}\text{Th}$  in turn decays with a half-life of 18.7 days to form  $^{223}\text{Ra}$  which decays via alpha emission with a half-life of 11.4 days.

(5 pts) Write a balanced reaction for the decay of  $^{227}\text{Ra}$  to  $^{227}\text{Ac}$ :



X 5

(E) (20 pts) If 20 ng of freshly isolated  $^{227}\text{Ra}$  is placed in solution, what will the mass of  $^{223}\text{Ra}$  in that solution be after 100 days.

mass $^{223}\text{Ra}$ = $3.80 \cdot 10^{-14} \text{g}$
--

$$20 \text{ ng} \cdot \frac{1 \text{g}}{10^9 \text{ ng}} \cdot \frac{1 \text{ mol}}{227 \text{ g}} \cdot \frac{6.02 \times 10^{23}}{1 \text{ mol}} = 5.304 \times 10^{12} = N$$

$$N_f = N_0 e^{-kt}$$

$$k = \frac{\ln 2}{47 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hour}} \cdot \frac{24 \text{ hours}}{1 \text{ day}} = 21.24 \text{ days}^{-1}$$

$$N = (5.304 \times 10^{12}) e^{-21.24 \cdot 100} = 0 \quad ^{227}\text{Ra}$$

$$N_{\text{Ac}} = N_0 \cdot ^{227}\text{Ra}$$

$$k_{\text{Ac}} = \frac{\ln 2}{21.8 \text{ years}} \cdot \frac{365 \text{ days}}{1 \text{ year}} = 8.711 \times 10^{-5} \text{ days}^{-1}$$

$$N_{\text{Ac}} = 5.304 \times 10^{12} e^{-8.711 \times 10^{-5} \cdot 100} = 5.258 \times 10^{12}$$

$$\therefore N_0 \text{ } ^{227}\text{Th} = 5.304 \times 10^{12} - 5.258 \times 10^{12} = 4.600 \times 10^6$$

$$k_{\text{Th}} = \frac{\ln 2}{18.7 \text{ days}} = 0.037 \text{ days}^{-1}$$

$$N_{\text{Th}} = 4.6 \times 10^6 e^{-0.037 \cdot 100} = 1129797040$$

$$\therefore N_0 \text{ } ^{223}\text{Ra} = 4.487 \times 10^6$$

$$k_{\text{Ra}} = \frac{\ln 2}{11.4 \text{ days}} = 0.0608 \text{ days}^{-1}$$

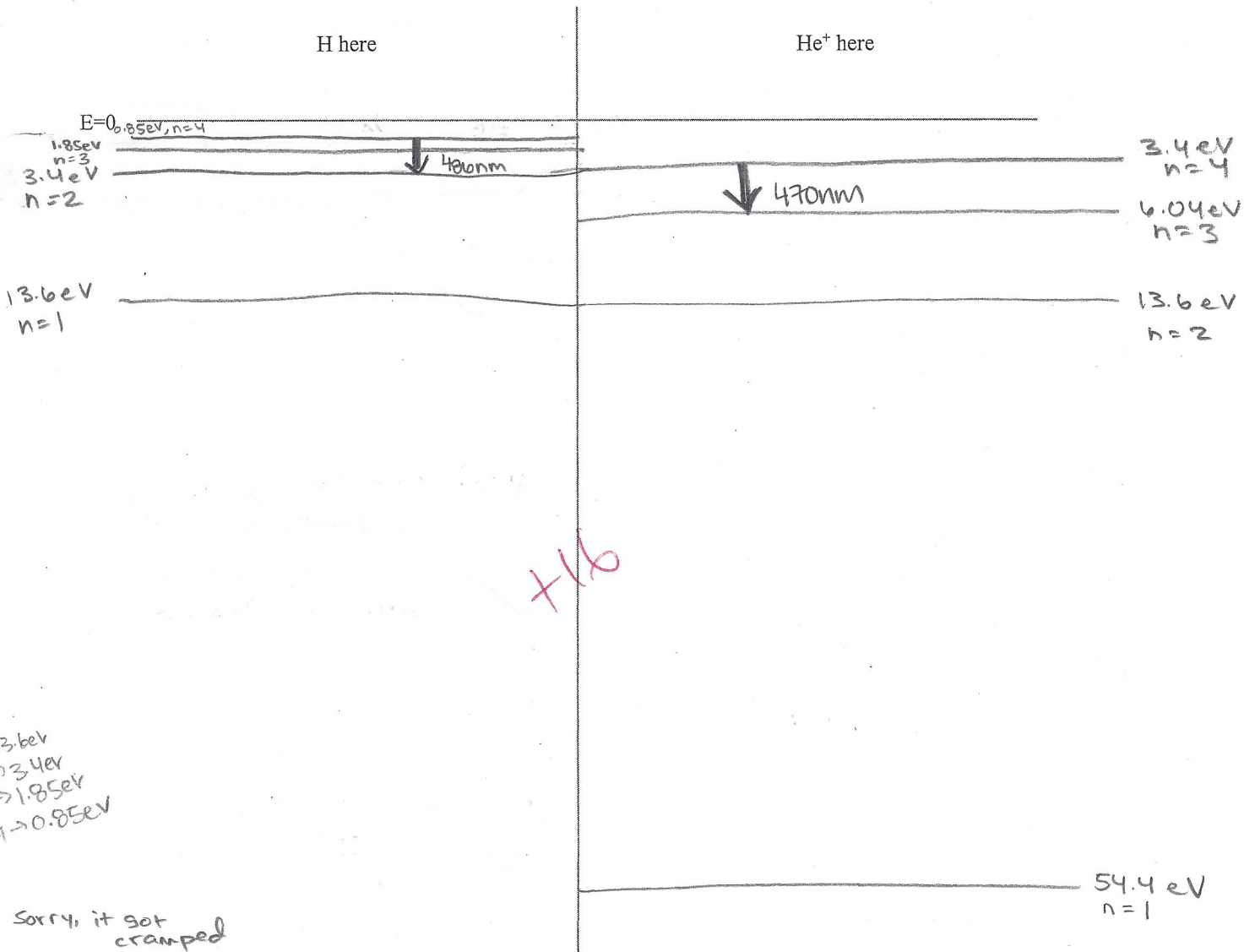
$$N_{\text{Ra}} = 4.487 \times 10^6 e^{-0.0608 \cdot 100} = 102653421.4$$

$$N_{\text{Ra}} \cdot \frac{1 \text{ mol}}{6.02 \times 10^{23}} \cdot \frac{223 \text{ g}}{1 \text{ mol}} = 3.80 \times 10^{-14} \text{ g} = 38.02 \text{ fg}$$

X 8

Section 5 (31 pts) Bohr Model

(A) (16 points) Side by side (and crudely to scale) sketch the lowest **four** energy levels of an H-atom and He<sup>+</sup> ion. Label each level with the quantum number "n" and the energy of the level. Keep the E=0 level constant for both.



(B) (5 points) Comment on similarities / differences. Explain the physical origin of the difference in ground state energy using a formula OTHER than the Bohr model formula (I know you have the potential to answer this correctly!)

Both have energy levels at 13.6eV and 3.4eV. He<sup>+</sup> has energy levels far higher than H does. He<sup>+</sup> has a far higher ground state energy than H because it has more protons in the nucleus and stronger nuclear force.

Coulomb potential =  $(9e9 \frac{Jm}{C^2}) \cdot \frac{q_1 q_2}{r}$  → (lower potential for larger radius)  
 Helium has higher potential energy that needs to be overcome to ionize?

(C) (10 pts) When excited with a high voltage electrical discharge, H and He gas both emit many sharp lines. The H spectrum contains a line at 486 nm, and the He spectrum contains a line at 470 nm. Explain both by labeling your sketch above.

H →  $\frac{hc}{\lambda} = E = \frac{1240eV \cdot nm}{486nm} = 2.551 eV$  → change from n=4 to n=2 (0.85eV - 3.4eV = 2.55eV)

He →  $E = \frac{1240eV \cdot nm}{470nm} = 2.63 eV$  → change from n=4 to n=3 (3.4eV - 6.04eV = 2.64eV)

Section 6. (28 pts) Particle in a Box:

(A) (8 points) Indicate for each graph whether the function drawn could be a valid wave-function for a particle in a box with walls at  $x=0$  and  $x=L$ . Explain BRIEFLY. (4 points each)

Graph	Valid / Reason
	<p>no <math>\psi^* \psi</math> not <del>continuous</del> <sup>continuous</sup>                      due to sharp changes                      in graph                      ✓</p>
	<p>yes, can be normalized                      is continuous                      0 @ <math>x=0, x=L</math>                      ✓</p>
	<p>no, not continuous                      ✓</p>
	<p><del>no</del> no, is not at 0 @  <math>x=0, x=L</math>                      ✓</p>

Section 6 - Particle in a Box

(B) (20 pts) The steady-state wave functions  $\Psi_n(x)$  and the energy for a particle in the  $n^{\text{th}}$  energy state in an infinite square box with walls at  $x=0$  and  $x=L$  are given by:  $\Psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$ ,  $E_n = \frac{n^2 h^2}{8mL^2}$ .

By explicit computation, determine the probability that the particle is found in the right-most 1/3 of the box for the 2<sup>nd</sup> excited state ( $n=3$ ).

$$(\Psi_n(x))^2 = \frac{2}{L} \sin^2\left(\frac{3\pi x}{L}\right)$$

$$\frac{2}{L} \int_{\frac{2L}{3}}^L \sin^2\left(\frac{3\pi x}{L}\right) dx$$

$$= \frac{2}{L} \int_{\frac{2L}{3}}^L (1 - \cos\left(\frac{6\pi x}{L}\right)) dx \quad \leftarrow \sin^2 u = \frac{1 - \cos 2u}{2}$$

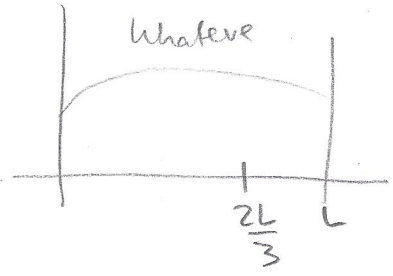
$$= \frac{1}{L} \left( x - \frac{L}{6\pi} \sin\left(\frac{6\pi x}{L}\right) \right) \Big|_{\frac{2L}{3}}^L$$

$$= \frac{1}{L} \left( L - \frac{L}{6\pi} \sin 6\pi - \left( \frac{2L}{3} - \frac{L}{6\pi} \sin 4\pi \right) \right)$$

$$= \frac{1}{L} \left( L - \frac{2L}{3} \right) = \left( 1 - \frac{2}{3} \right) = \frac{1}{3} = 0.333$$

WTF.

0.333



check according to calculator  $2 \int_{2/3}^1 (\sin(3\pi x))^2 dx = 0.333$



**Integrated Rate Laws:**  
 1st:  $\ln[A] = \ln[A_0] - kt$   
 2.  $\frac{1}{[A]} = \frac{1}{[A_0]} + 2kt$   
 0:  $[A] = [A_0] - kt$

**Michaelis-Menton:**  
 $E + S \xrightleftharpoons[k_2]{k_1} ES \xrightarrow{k_{cat}} E + P$   
 E = free enzyme  
 ES = enzyme-substrate complex  
 When  $[S] \gg K_m$ ,  $k_1[S] \gg k_2$ ,  $k_1[E] \approx -d[ES]/dt$   
 $\frac{d[ES]}{dt} = 0 = \text{steady state approx}$   
 $K_m = \frac{k_1 + k_2}{k_1}$   
 $[ES] = \frac{[E][S]k_1}{K_m + [S]}$   
 $\text{rate} = \frac{V_{max}[S]}{K_m + [S]}$   
 graph: rate vs [S] showing a hyperbolic curve.  $K_m = \frac{V_{max}}{2}$

**PHYSICS**  
 $E = ma$   
 $KE = \frac{1}{2}mv^2$   
 $p = mv$   
 $G = \frac{GMm}{r^2}$   
 Normal units  
 $IN = kg \cdot m^2/s^2$   
 $J = Nm$   
 $e^- \text{ charge} = 1.6e-19C$   
 $v = \lambda/\lambda$   
 $W = J/s$   
 $F = 96485 C/mol$   
 $eV = 1.602e-19J$   
 $Bq = s^{-1} = Hz$   
 $Gy = J/kg = 100rad = dose$   
 $Sv = 100rem$   
 $LD_{50} = 5 Sv$   
 $bkq \cdot rad = 2mSv$

**Radioactivity**  
 $m_p = 1.6726e-27kg$   
 $= 938 MeV/c^2$   
 $m_n = 1.6749e-27kg$   
 $= 939.5 MeV/c^2$   
 $m_e = 9.109e-31kg$   
 $= 0.511 MeV/c^2$   
 $u = 931.494 MeV$

**Quantum Mech**  
 $hc = 1240 eV \cdot nm$   
 $h = 6.626e-34 Js$   
 Rydberg =  $2.18e-18J$   
 $\hbar = h/2\pi$   
 $a_0 = 0.529 \text{ \AA}$   
 $E_0 = 8.854e-12 C^2$

**Conversions**  
 $Meq = 1e6$   
 $milli = 10^{-3}$   
 $micro = 10^{-6}$   
 $nano = 10^{-9}$   
 $pico = 10^{-12}$   
 $1 \text{ \AA} = 10^{-10}$   
 $Tera = 10^{12}$   
 $Giga = 10^9$   
 $kilo = 10^3$   
 $femto = 10^{-15}$

**Quality factors**  
 $\alpha 1-20$   
 $\beta = 1$   
 $\gamma, xray = 1$   
 $n^0 = 3-10$   
 $P^+ = 1-10$   
 $1 Sv = 1 J/kg \cdot QF$

**NUCLEAR CHEMISTRY**  
 $E = mc^2$   
 $E^2 = m^2c^4 + p^2c^2$   
 $\therefore \text{PHOTONS } E = pc$   
**Nuclear Binding**  
 $BE = (Zm_p + Nm_n - m_a)c^2$   
 $V = 9.7 \cdot 10^2 \cdot \frac{Z^2}{4\pi \epsilon_0 r}$   
 $\Delta m = m[{}^{14}_6B] + 2m[e^-] - m[{}^{14}_6C]$   
 ${}^{14}_6C \rightarrow {}^{14}_5B + e^- + \bar{\nu}$   
 ${}^{231}_{91}Pa + e^- \rightarrow {}^{231}_{90}Th + \nu$   
 $p^+ + e^- \rightarrow n^0 + \nu_e$   
 gamma decay - Pb  $\rightarrow$  gives off EM rad as photon

**Nuclear decay**  
 alpha decay - paper  
 ${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^4_2He$   
 beta decay - Al  
 ${}^{14}_6C \rightarrow {}^{14}_7N + e^- + \bar{\nu}$   
 positron emission  
 ${}^{11}_6C \rightarrow {}^{11}_5B + e^+ + \nu$   
 electron capture  
 ${}^{231}_{91}Pa + e^- \rightarrow {}^{231}_{90}Th + \nu_e$

**Kinetics**  
 $A = -\frac{dN}{dt} = kN$   
 $A = A_0 e^{-kt}$   
 $k = \ln 2 / t_{1/2}$   
 $N = N_0 e^{-kt}$   
 $K_{tot} = k_{bio} + k_{rad}$   
**conservation**  
 - energy  
 - charge  
 - momentum  
 - lepton #  $\nu_e, \bar{\nu}_e$   
 - baryon #

**Fission**  
 ${}^1_0n + {}^{235}_{92}U \rightarrow {}^{142}_{54}Xe + {}^{92}_{38}Kr + 2{}^1_0n$

**Fusion**  
 ${}^2_1H + {}^3_1H \rightarrow {}^4_2He + e^- + \bar{\nu}$   
 ${}^2_1H + {}^2_1H \rightarrow {}^3_2He + \gamma$   
 ${}^3_2He + {}^3_2He \rightarrow {}^4_2He + 2{}^1_1H$

**EM rad**  
 $E(x,t) = E_{max} \cos(2\pi(\frac{x}{\lambda} - \nu t)) - \nu t$   
 - constructive/destructive interference  
**Light absorption**  
 $\nu = E + -e_0/h$  or  $\Delta E = h\nu$

**BOHR MODEL**  
 $E = \frac{1}{2}mv^2 - \frac{Ze^2}{4\pi\epsilon_0 r} = \text{kinetic} + \text{potential}$   
 Ang. momentum =  $L = mvr = n\hbar/2\pi$   
 $r_n = \frac{n^2 a_0}{Z}$   
 $v_n = \frac{nh}{2\pi m_e r_n} = \frac{Ze^2}{2\epsilon_0 nh}$

**Atomic Spectra interpretation**  
 $\nu = (3.29e15 s^{-1}) Z^2 (\frac{1}{n_1^2} - \frac{1}{n_2^2})$

**Photoelectric effect**  
 $E = h\nu$   
 $E_{max} = \frac{1}{2}mv^2 = h\nu - \phi = hc/\lambda - \phi$   
 $\phi = h\nu_0$  energy barrier must overcome

**Schrodinger**  
 $\psi^* \psi = \text{probability}$   
 particle in a small vol  $dV$  @  $(x,y,z)$  @ time  $t$

**1D**  
 $\psi(x) = A \sin kx$   
 $\frac{d^2\psi(x)}{dx^2} = -(\frac{h^2 E}{2m}) \psi(x)$   
 $E_n = \frac{n^2 h^2}{8mL^2}$   $n = 1, 2, 3, \dots$   
 $E = \frac{h^2}{8m} (\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2})$

**Particle in a Box (med's to die)**

**Quantum mech**  
 waves  $\psi$   
 travelling  $k = 2\pi/\lambda$  speed =  $\lambda\nu$   
 $\psi(x,t) = A \sin(\omega t - kx)$   $c = \lambda\nu$  photons  
 $\nu = \frac{c}{\lambda} \therefore E = h\nu = \frac{hc}{\lambda}$   
 standing  
 $\psi(x,t) = 2A \sin(kx) \cos(\omega t)$

**Energy quantization**  
 $P(\lambda) = \frac{8\pi hc}{\lambda^5} \cdot \frac{1}{e^{hc/\lambda kT} - 1}$   
 UV catastrophe = classical  
 Planck  
 - E = discrete, gain/lose  $\Delta E$  in quanta  
 - to emit E, T sufficiently high  
**Frank-Hertz and Bohr**  
 $\nu = \Delta E/h = \frac{eV_{thr}}{h}$   
 $E_n = \frac{2^2}{n^2} \text{ Ryd} = 13.6 \frac{Z^2}{n^2} eV$

**Ratio of Decay**  
 $\frac{N_{1t}}{N_{10}} = e^{-\lambda t/kT}$   $\Delta E = 136eV$  (ionization)

**Double slit exp.**  
 $\frac{n\lambda_1}{n\lambda_2} = \frac{d \sin \theta_1}{d \sin \theta_2}$   $\frac{\lambda_1}{\lambda_2} = \frac{d_1}{d_2}$   
 $\frac{E_2}{E_1} = \frac{d_1}{d_2}$  bc  $E \propto \frac{1}{\lambda}$

**PROBABILITY**  
 must be normalized  $\int \psi(x)^2 dx = 1$   
 $\psi$  must be cont  
 - vertical line test  
 Normalized  
 $\int_a^b \frac{2}{L} \sin^2(\frac{2\pi x}{L}) dx$   
 $= \frac{2}{L} (\frac{x}{2} - \frac{L}{4\pi} \sin(\frac{4\pi x}{L})) \Big|_a^b$

**De Broglie waves (standing wave)**  
 $\lambda = h/mv = \frac{h}{p}$

**Heisenberg kms**  
 $\Delta x \Delta p \geq h/4\pi$   
 max v of  $pe @ 12.4nm$   
 $E = hc/\lambda$  then  $E = \frac{hc^2}{\lambda^2} = 13.6eV$   
 $E_{ke} = E_{photon} - E_{ionization}$

**Gamma**  $1.2 \text{ pm}$   
**X ray**  $0.0001 \text{ nm}$   
**UV**  $10 \text{ nm}$   
**VIS**  $400 \text{ nm}$  (violet) to  $700 \text{ nm}$  (red)  
**IR**  $1000 \text{ nm}$   
**radio**  $0.01 \text{ cm}$  to  $7.5 \text{ m}$

**Leptons**

charge	lepton #
$e^-, \mu^-, \tau^-$	-1
$e^+, \mu^+, \tau^+$	+1
$\nu_e, \nu_\mu, \nu_\tau$	+1
$\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau$	-1

**Baryons**

charge	baryon #
$p^+$	+1
$n^0$	0
$\bar{p}^-$	-1
$\bar{n}^0$	0

**Finding min  $\lambda$**   
 $KE = h\nu - \phi = \frac{1}{2}mv^2$   $\lambda = \frac{h}{p}$

**Diagram**

**Diagram**

**Diagram**